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Provincial Department of Education - NWP

10 E I

First Term Test - Grade 13 - 2020

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
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For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

01. Find the partial fractions of $\dfrac{4x^3+10x+4}{x(2x+1)}$.

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02. Solve the inequality $|x-3| > \dfrac{1}{2}x$, using graphs. **Hence or otherwise**, find the range of the values of x which satisfies the inequality $2|x+4| > x+7$.

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03. The polynomial $f(x) = x^4 + 3x^3 + ax + b$ where, a and b are constants. When $f(x)$ is divided by $x^2 + x - 1$, the remainder is $2x + 3$. Find the values of a and b.

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04. Find the value of $\lim_{x \to \pi} \dfrac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$.

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05. A curve C has parametric equations $x = 6\cos 2t$, $y = 2\sin t$, $\left(-\dfrac{\pi}{2} < t < \dfrac{\pi}{2}\right)$. Show that $\dfrac{dy}{dx} = \lambda \operatorname{cosec} t$, giving the exact value of the constant λ. Find the equation of the normal to C at the point where $t = \dfrac{\pi}{6}$.

06. When $y = x^2 - 2x + 5$, find the equation of the tangent line at the point $x = 2$

07. Find the area above x - axis, bounded by the line $x = 4$ and the curve $y = f(x)$ where

$$f(x) = \begin{cases} x^2 & ; 0 \le x \le 1 \\ \sqrt{x}; & ; x \ge 1 \end{cases}$$

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08. A straight line through the point $(1, 1)$ meets the x - axis A and the y - axis at B. find the equation of the locus of the midpoint of AB.

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09. Find the equations of the line through the points $(3, 2)$ and making an angle of 45° with the line $x-2y=3$

10. Solve the equation $\tan^{-1}\left(\frac{1}{x-1}\right)-\tan^{-1}\left(\frac{1}{x+1}\right)=\tan^{-1}\left(\frac{1}{2}\right)$ where ; $x > 0$, $x \neq -1$

Combined Mathematics - Grade 13 - I (Part B)

❖ *Answer only five questions.*

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11. *a.* The quadratic equation $x^2-2x+3=0$ has roots α and β. Without solving the equation, write down the values of $\alpha+\beta$ and $\alpha\beta$.

    Hence,

    i. show that $\alpha^2+\beta^2=-2$.

    ii. find the value of $\alpha^3+\beta^3$.

    iii. show that $\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2$.

    iv. find the quadratic equation which has the roots $\alpha^3-\beta$ and $\beta^2-\alpha$ giving your answer in the form $px^2+qx+r=0$.

    b. If $x-2$ is a factor of polynomial $P(x)=3x^3+Cx^2+x-2$, find the value of C. For this value of C,

    i. find the remaining factor of $P(x)$.

    ii. show that $P(x)=0$ has only one real root.

    iii. if $P(x)+\lambda(x-2)(3x-1)=0$ has 3 distinct real roots, find the range of the values of λ.

12. *a.* If a, b, c are real constants, $f(x)=ax^2+2bx+c$ and $g(x)=2(ax+b)$" write the discriminant of the quadratic expression $F(x)\equiv f(x)+\lambda g(x)$; where, λ is a real constant.

    If the roots of $f(x)=0$ are real and distinct, deduce that the roots of $F(x)=0$ too are real and distinct .

    b. *i.* If the set of values of x which satisfies the inequality $5|x-1|+3\leq ax+b$ is represented by $\{x \mid 0\leq x\leq 5, x\in R\}$, find the constants a and b .

    ii. Find all real values of x such that $x^2-2x-24<0$ and $x^2+2x>0$.

    c. Show that $\log_a c=\dfrac{\log_b c}{\log_b a}$, when $a, b, c\in\Re$. Hence deduce that $\log_a c=\dfrac{1}{\log_c a}$.

    If $x=\log_{2a} a,\ y=\log_{3a} 2a,\ z=\log_{4a} 3a$, show that $xyz+1=2yz$.

13. *a.* y is a function of x and $x = \tan\theta$. Express $\dfrac{d^2y}{dx^2}$, in terms of $\dfrac{dy}{d\theta}$ and $\dfrac{d^2y}{d\theta^2}$.

If $(1+x^2)^2\dfrac{d^2y}{dx^2} + 2x(1+x^2)\dfrac{dy}{dx} + y = 0$, then show that $\dfrac{d^2y}{d\theta^2} + y = 0$.

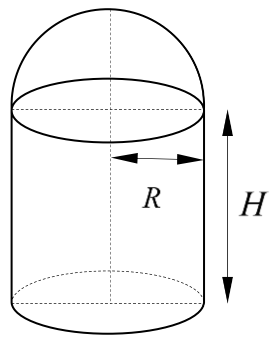
b. Show that the area bounded by the parabolas $y^2 = 5x+6$ and $x^2 = y$ is $5\frac{2}{5}$ square units.

Find the volume of the solid generated by rotating the area bounded by the above parabola $y^2 = 5x+6$ and the line $x = 1$ about the x - axis.

14. *a.* Let $f(x) = \dfrac{x+4}{(x-1)^2}$ for $x \neq 1$ and $x \in R$. Find the first derivative of $f(x)$. It is given that the second derivative of $f(x)$, $f''(x)$ as $f''(x) = \dfrac{2(x+14)}{(x-1)^4}$.

Draw a rough sketch of the graph $y = f(x)$, representing turning points, asymptotes and inflection points.

b. A storage unit has the shape of a hemisphere on top of a cylinder and a fixed volume V. Determine the dimensions of the solid (the radius R and the height H) that m inimize the total area.



15. *a.* Using partial fractions, find $\displaystyle\int \frac{1}{(x^2-1)(2x+1)}\,dx$.

Hence, using a suitable substitution, find $\displaystyle\int \frac{1}{\sin\theta + \sin 2\theta}\,d\theta$

b. Using integration by parts, find $\int e^{5x}\sin 2x\,dx$.

c. Show that $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x.\cos x}{\cos^2 x + \sin^4 x}\,dx = 2\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos^2 2x + 3}\,dx$.

Hence, using the substitution $z = \cos 2x$ or otherwise, show that $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x.\cos x}{\cos^2 x + \sin^4 x}\,dx = 2\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos^2 2x + 3}\,dx = \frac{\pi}{6\sqrt{3}}$.

16. *a*. Show that $a_1x+b_1y+c_1+\lambda(a_2x+b_2y+c_2)=0$ is a straight line which passes through the meeting point of the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, where λ is a parameter.

$ABCD$ is a parallelogram. Equations of the lines AB, BC, CD and DA, are $y=m_1x+c_1$, $y=m_2x+d_1$, $y=m_1x+c_2$ and $y=m_2x+d_2$ respectively. Without finding the coordinates of the vertices,

i. find the equations of the diagonals AC and BD.

ii. If $ABCD$ is a rhombus, then show that $(1+m_1^2)(d_2-d_1)^2=(1+m_2)^2(c_2-c_1)^2$.

b. A line cuts the x - axis at $A(7,0)$ and the y - axis at $B(0,-5)$. A variable line PQ is drawn perpendicular to AB cutting the x - axis in P and the y - axis in Q. If AQ and BP intersect at R, find the equation of the locus of R.

17. *a*. Using the expansion for, show that $\tan 15^\circ = 2-\sqrt{3}$.

Show that $\tan\left(\frac{x}{2}\right)=\frac{\sqrt{1+\tan^2 x}-1}{\tan x}$ for $0<x<\frac{\pi}{2}$.

Show that $\tan 7\frac{1}{2}^\circ = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$ and deduce that $\cot 7\frac{1}{2}^\circ = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$.

b. Find the general solution of the equation $\cot x+\cos ecx=\sqrt{3}$.

c. State and prove the **sine rule** for any triangle ABC in the usual notation. Using the sine rule prove that

i. $(b-c)\cos\left(\frac{A}{2}\right)=a\sin\left(\frac{B-C}{2}\right)$

ii. $\frac{a+b-c}{a+b+c}=\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)$

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Provincial Department of Education - NWP

10 E II

First Term Test - Grade 13 - 2020

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

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  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
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(10) Combined Mathematics II		
Part	Question No	Marks Awarded
A	1	
	2	
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	4	
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	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
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	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

(Part - A)

01) A particle is projected under gravity from a point O, on the floor with velocity u making an angle θ with horizontal. Draw velocity time graphs separately for both horizontal and vertical motions of the particle, until it hits a point on the horizontal plane through the point O. Hence show that the time taken for the particle to reach the greatest height and to hit the ground again is equal, if it hits the ground with the same projected velocity. Also show that it equals $\frac{u\sin\theta}{g}$ and the horizontal range of the particle is $\frac{2u^2}{g}\sin\theta\cos\theta$. where g is the gravitational acceleration

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02) A particle P of mass m, moving along a smooth horizontal table with velocity u, collides directly with a particle Q of equal mass, which is at rest on the table. If the coefficient of restitution for the collision is e. Show that the velocities of the particles P and Q after the collision are $\frac{u}{2}(1-e)$ and $\frac{u}{2}(1+e)$ respectively. Show that the loss of energy due to collision is $\frac{mu^2}{4}(1-e^2)$.

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03) A particle is projected under gravity with initial velocity u, at an angle θ to the horizontal. Show that the maximum height reached by the particle is $\frac{u^2}{2g}\,sin^2\,\theta$. When the particle reaches half of the maximum height of its path, if the direction of the motion of the particle makes an angle of 30^0 with the downward vertical. Show that $\theta = tan^{-1}\left(\sqrt{6}\right)$.

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04) If A motor car of mass M metric tons, with maximum power H kilowatts, moves downwards along the line of greatest slope of a level road of inclination 30^0 to the horizontal with the constant velocity $v\;ms^{-1}$ under a resistance R Newtons, show that the resistance on the car is $R = \left[\frac{2H+Mgv}{2V}\right]10^3$ Newtons. Subsequently, at the same instant which the car is entered to a horizontal road with the same constant resistance, if the engine of the car is stopped, show that the retardation on the car is, $\frac{1}{M}\left[\frac{MgV+2H}{2V}\right]$.

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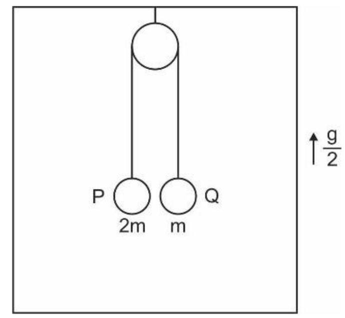
05) Two particles P and Q of masses $2m$ and m respectively are attached to the two ends of a light inextensible string passing around a smooth light pulley which is fixed in the ceiling of a lift moving upwards with an uniform acceleration of $\frac{g}{2}$ and the system is released gently for motion. show that the acceleration of each particle relative to the lift is $\frac{g}{2}$ and the tension in the string is $2mg$ where g is the gravitational acceleration.



06) $\underline{a}$, $\underline{b}$ and $\underline{c}$ are three non zero, non parallel vectors. $|\underline{a}| = 1$, $|\underline{b}| = 3$ and $|\underline{c}| = 2$. Let $\underline{c} = \underline{a} + \lambda\underline{b}$, $\underline{a}$ and $\underline{b}$ are perpendicular to each other and the angle between $\underline{b}$ and $\underline{c}$ is $\frac{\pi}{3}$.

i. Find the value of λ.

ii. Also show that the angle between $\underline{a}$ and $\underline{c}$ is $\frac{\pi}{3}$.

07) A thin uniform weighted rod AB is kept in a vertical plane perpendicular to a wall such that the end B is against the smooth vertical wall and the end A is on a rough horizontal floor making an angle θ with the floor. The coefficient of friction between the rod and the floor is $\frac{1}{2}$. Show that $\theta \geq \frac{\pi}{4}$ for the rod to be in equilibrium.

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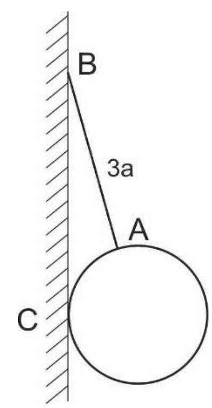
08) A point C on a sphere of radius a and weight w rests in contact with a smooth vertical wall and is supported by a light inextensible string of length $3a$ joining a point A on the sphere to a point B on the wall. B lies vertically above C. Find the tension in the string and the normal reaction on the sphere by the wall, in terms of w.



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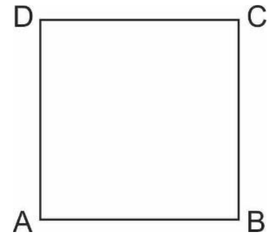
09) The forces with magnitude $9P, 7P, 2P, P$ and $3\sqrt{2}\ P$ Newtons act along the sides AB, CB, CD, AD and BD in a square ABCD, in the direction of the order of the letters respectively. Find the magnitude and direction of the resultant of the system of forces.



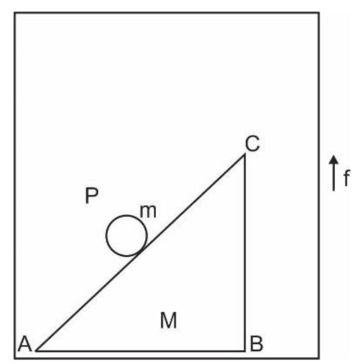
10) One end of a light inextensible string of length $3r$ is attached to a fixed-point A and a particle of mass m is attached to the other end. The particle moves with velocity v in a horizontal circle of radius r and center O, which lies vertically below A. Show that $v^2 = \frac{gr}{2\sqrt{2}}$. where g is the gravitational acceleration.

Combined Mathematics 13 – II (Part - B)

11) (a) At $t = 0$, A, balloon B is released from a point O which is on the floor, such that it moves vertically upwards with a constant acceleration f. After another time $t = T$, from the same point O, a ball A is projected vertically upwards under the gravitational acceleration g with velocity u $(g > f)$. Draw velocity – time graphs for the motion of the balloon and the ball in the same diagram. Hence if the ball meet the balloon at its maximum height, show that $u = \frac{gT\sqrt{f}\,[\sqrt{g} + \sqrt{f}]}{g-f}$. Further when the ball hit the horizontal floor through the point O, show that the height of the balloon is, $\frac{f}{2}\left(T + \frac{2u}{g}\right)^2$.

(b) A helicopter flies from an aircraft carrier ship moving with constant velocity u towards North, to an island which is situated at a distance $d\ km$ due east and perpendicular to the path of the ship and returns to the ship. (it doesn't take an extra time to turn the helicopter) Furthermore both motions along two straight paths are at the same height and with the same velocity $v,\ (v > u)$. Also in the motions of flying to the island and returning to the ship, if the helicopter flies in a direction θ East of North and from East of North respectively, relative to ship. Draw the velocity triangles for both cases in the same diagram. Show that the total time taken for the helicopter to fly from the ship to the island and the time taken to return the ship is $\frac{2d\sqrt{V^2 - u^2 \sin^2 \theta}}{(V^2 - u^2) sn\theta}$.

12) A wedge of mass M with smooth faces and a triangular vertical cross section ABC is kept on a smooth horizontal floor of a lift moving vertically upwards with constant acceleration F. A particle P of mass m is kept on the inclined plane which makes an angle α to the horizontal and the system is released gently for motion such that the particle moves along the inclined plane and the wedge moves along the horizontal floor of the lift. Applying the equations for the particle P in the direction CA, and for the system in the direction AB, show that,



$i.$ The acceleration of the wedge relative to the lift is $\frac{m(F+g)\sin\alpha\cos\alpha}{M+m\sin^2\alpha}$

$ii.$ The acceleration of the particle relative to the wedge is $\frac{(M+m)(F+g)\sin\alpha}{M+m\sin^2\alpha}$

$iii.$ If the lift moves with uniform velocity or is at rest deduce that the acceleration of the wedge relative to the lift is $\frac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha}$ and deduce that the acceleration of the particle relative to the wedge is, $\frac{(M+m)g\sin\alpha}{M+m\sin^2\alpha}$

Also at this time show that the acceleration of the particle relative to the earth is,

$$\frac{g\sin\alpha}{M+m\sin^2\alpha}\,[m^2\cos^2\alpha + (M+m)^2 - 2m(M+m)\cos\alpha]^{\frac{1}{2}}$$

$iv.$ If the lift moves vertically downwards with constant acceleration g, deduce that the wedge is at rest relative to the lift or the particle moves with uniform velocity relative to the wedge. where g is the gravitational acceleration.

13) A particle P of mass m suspends freely from a point O, using a light inextensible string of length l. Then it is projected with a horizontal velocity u.

$i.$ Show that the velocity v of the particle, when OP makes an acute angle θ with the downward Vertical, gives by $v^2 = u^2 + 2gl\, cos\,\theta - 2gl$.

$ii.$ Show that the tension of the string, gives by $T = \frac{m}{l}(u^2 + 3gl\cos\theta - 2gl)$.

$iii.$ If the particle travels the entire circular path in a vertical plane, show that $u^2 > 5gl$.

$iv.$ If the particle leaves the circular path without travailing the entire circular path, show that $2gl < u^2 < 5gl$.

$v.$ Initially if the particle is projected with velocity $2\sqrt{gl}$, show that the velocity of the particle is $\sqrt{2gl}$, when the string reaches the horizontal position through O.

$vi.$ When the string reaches the horizontal position through O, the string touches a smooth peg A which is at a horizontal distance $\frac{l}{2}$ from O and the particle moves in a vertical circular path with centre as the peg A, show that the particle travels the complete vertical circular path with center A.

14) (a) The position vectors of the points A, B and C relative to the origin O are $2\,\underline{i} + 3\,\underline{j}$, $\underline{i} + 4\,\underline{j}$ and $4\,\underline{i} - \underline{j}$ respectively. The points P, Q and R lie on the sides BC, CA and AB such that $BP{:}PC = 3{:}1$, $CQ : QA = 1{:}2$ and $AR : RB = 2{:}1$. Show that,

$i.$ RQ is parallel to BC

$ii.$ $\overrightarrow{AP} = \frac{5}{4}\,\underline{i} - \frac{11}{4}\,\underline{j}$ and $\overrightarrow{BQ} = \frac{7}{3}\,\underline{i} - \frac{11}{3}\,\underline{j}$

$iii.$ If the straight lines AP and BQ intersect at S, find the ratios $AS{:}SP$ and $BS{:}SQ$.

(b) A system of coplanar forces consisting of four forces is given below.

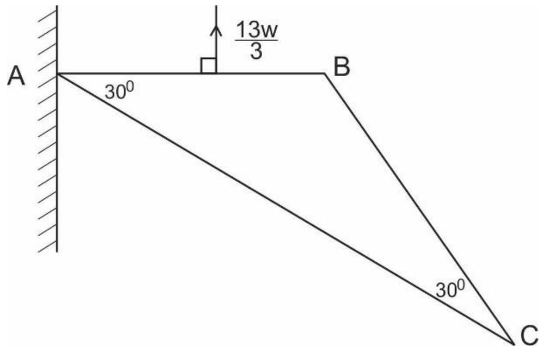
Point	Position Vector	Force
A	$3\,\underline{i} - \underline{j}$	$\underline{i} - 4\,\underline{j}$
B	$2\,\underline{i} + 2\,\underline{j}$	$3\,\underline{i} + 6\,\underline{j}$
C	$-\underline{i} - \underline{j}$	$-9\,\underline{i} + \underline{j}$
D	$-3\,\underline{i} + 4\,\underline{j}$	$5\,\underline{i} - 3\,\underline{j}$

where $\underline{i}$ and $\underline{j}$ have their usual meanings.

$i.$ Show that the system of forces reduces to a couple of forces and find the magnitude of the moment of that couple and the sense of it.

$ii.$ When the force acting at the point D is removed and the force acting at the point A is replaced to the point $\underline{i} - 8\,\underline{j}$, show that the system reduces to a single force and find its magnitude and direction.

15) (a) Three uniform rods AB, BC and AC of weights w, w and $2w$ respectively are smoothly jointed at their ends, A, B and C to form the framework in the shape of a triangle ABC. As shown in the figure, it is hinged to a smooth vertical wall at A and is kept in equilibrium in a vertical plane such that AB is horizontal by applying a horizontal force of $\frac{13\,w}{3}$ on the rod AB at D.



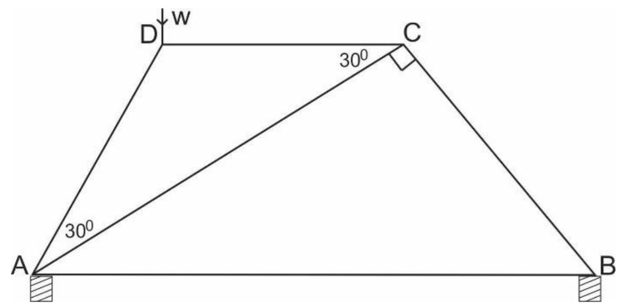
$AB = BC = 2a$ හා $B\hat{A}C = A\hat{C}B = 30^0$

(i) Show that $AD = \frac{3a}{2}$.

(ii) Find the horizontal and vertical components of the reactions acting at the joints B and C.

(b) The frame work shown in the adjoining figure is made of smoothly joining five light rods AB, BC, AC, AD and CD at their ends. A load w is applied at the joint D. The framework is kept on two smooth supports at A and B and is in equilibrium in a vertical plane such that AB and DC are horizontal.

$D\hat{A}C = D\hat{C}A = 30^0$, $A\hat{C}B = 90^0$



(i) Find the reactions on the framework by the supports A and B.

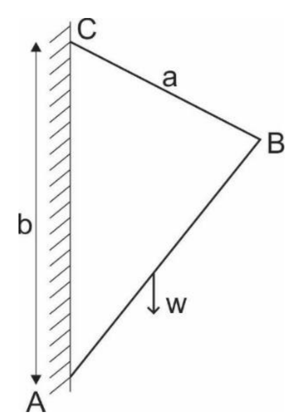
(ii) Using the Bow's notation draw a stress diagram and hence find the stresses along the five rods classifying them in to tensions or thrusts.

16) (a) One end A of an uniform ladder AB of weight w and length $2a$ is placed on a rough horizontal floor and the other end B is in contact with a rough vertical wall. The coefficient of friction at both ends of the ladder is $\frac{2}{3}$. The ladder is held in a vertical plane perpendicular to the wall such that the inclination of the ladder to the floor is $\tan^{-1}\left(\frac{3}{4}\right)$. A child whose weight is $2w$ can go along the ladder to a distance x starting from A. Show that the ladder attains the limiting equilibrium when .$x = \frac{19\,a}{13}$.

(b) An uniform rod AB of weight w and length a is hinged to a vertical wall at A and maintains in equilibrium by a string of length a which is attached to B and to a point C vertically aboove A which is at a distance b from A. Show that the,

(i) tension in the string is $\frac{wa}{2b}$ and

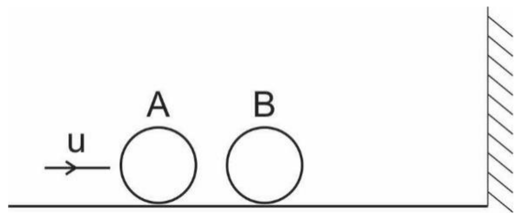
(ii) reaction of the hinge A is, $\frac{(\sqrt{a^2 + 2b^2})w}{2b}$



17) (a) The total mass of a child with his bicycle is $95kg$. The child rides his bicycle with maximum velocity of $3\ ms^{-1}$ on a horizontal road and he rides his bicycle up a road of inclination 30^0 to the horizontal, with a maximum velocity of $2\ ms^{-1}$. If he rides his bicycle downwards along the same inclined road, then find the resistance to the motion and the acceleration. When the velocity is $4\ ms^{-1}$.

Assume that for all motions the child rides his bicycle with the same power and the resistance to motion is proportional to the square of its velocity. (Let $g = 10\ ms^{-2}$)

(b)



Two smooth spheres A and B with same radii and each of mass $2m$ and $7m$ are kept on a smooth horizontal plane against a smooth vertical wall. A is projected with velocity u to strike B directly. The coefficient of restitution between A and B is $\frac{1}{2}$.

(i) Find the velocities of A and B after the collision.

(ii) After the collision of A and B the sphere B collides with the wall and rebounces. If the coefficient of restitution between the sphere B and the wall is $\frac{1}{3}$, show that the spheres A and B will not collide again.

(iii) Show that the loss of kinetic energy of the system, due to collision is $\frac{301}{324}\ mu^2$.

Grade 13 - First Term Test 2020
Combined Mathematics I
Marking Scheme

(01)

$$\frac{4x^3+10x+4}{x(2x+1)} = Ax+B+\frac{C}{x}+\frac{D}{2x+1} \quad (10)$$

$$4x^3+10x+4 \equiv (Ax+B)x(2x+1)+C(2x+1)+Dx$$

$$\equiv (Ax+B)(2x^2+x)+C(2x+1)+Dx$$

$x^3 \rightsquigarrow \quad 2A=4$

$A=2$

$x^2 \rightsquigarrow \quad A+2B=0$

$B=-1$

(15)

$x \rightsquigarrow \quad B+2C+D=10$

$2C+D=11$

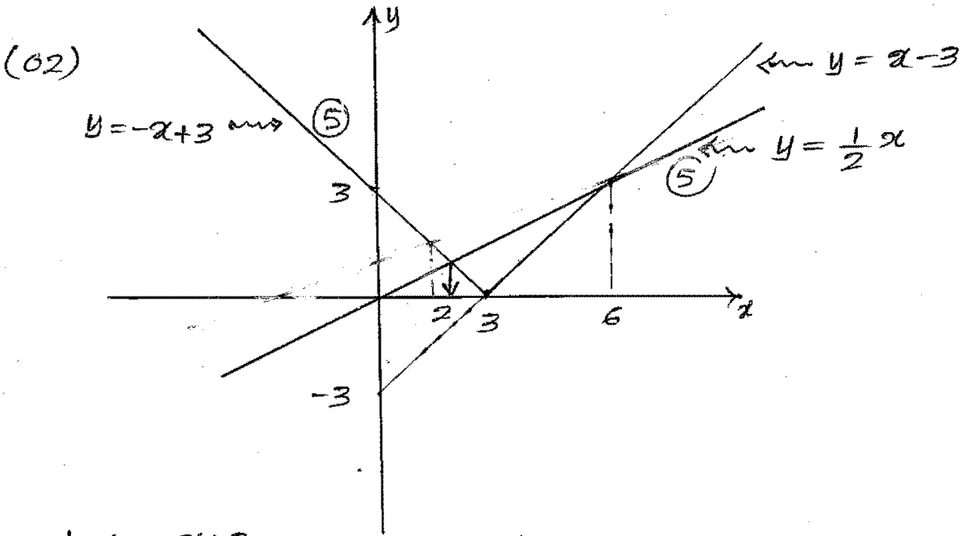
constant $\rightsquigarrow \quad C=4$

$D=3$

$$\frac{4x^3+10x+4}{x(2x+1)} = \underline{\underline{2x-1+\frac{4}{x}+\frac{3}{2x+1}}}$$

$\boxed{25}$

(02)



$\frac{1}{2}x = -x + 3$

$\frac{3}{2}x = 3$ (5)

$x = 2$

$x - 3 = \frac{1}{2}x$

$x = 6$

$\underline{\underline{x \in (-\infty, 2) \cup (6, \infty)}}$ (5)

$|x - 3| > \frac{1}{2}x$

Let $x = t + 7$

$\Rightarrow 2|t + 4| > t + 7$

$\Rightarrow 2|x + 4| > x + 7$

$\underline{\underline{x \in (-\infty, -5) \cup (-1, \infty)}}$ (5)

$\boxed{25}$

$$= \lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right)^2 \times \lim_{\theta \to 0} \frac{1}{(1+\cos\theta)\{\sqrt{2-\cos\theta}+1\}}$$ 5

$$= 1^2 \times \frac{1}{2\times 2}$$ 5

$$= \frac{1}{4}$$ 5

$\boxed{25}$

(05) $x = 6\cos 2t$ $\qquad y = 2\sin t$

$$\frac{dx}{dt} = -12\sin 2t \quad \text{5} \quad \frac{dy}{dt} = 2\cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{2\cos t}{-12\sin 2t}$$

$$= -\frac{1}{12}\frac{\cos t}{\sin t\cos t}$$

$$= -\frac{1}{12}\operatorname{cosec} t \qquad \lambda = -\frac{1}{12}.$$ 5

$$C \equiv \left(6\cos\tfrac{\pi}{3},\ 2\sin\tfrac{\pi}{6}\right)$$

$$= (3, 1)$$ 5

$$m_1 = \left(\frac{dy}{dx}\right)_{t=\pi/6} = -\frac{1}{12}\operatorname{cosec}\frac{\pi}{6}$$

$$= -\frac{1}{12}\times 2$$

$$= -\frac{1}{6}$$

$$m_1 m_2 = -1 \Rightarrow m_2 = 6$$ 5

$\therefore$ Eq^n of the normal

$$\frac{y-3}{x-1} = 6$$

$$y - 6x + 3 = 0$$ 5

$\boxed{25}$

3

(03) $f(x) = x^4 + 3x^3 + ax + b$

$f(x) = (x^2+x-1)(x^2+Ax+B) + 2x+3$ (10)

$x^3 \rightsquigarrow A + 1 = 3$

$\underline{\underline{A = 2}}$

$x^2 \rightsquigarrow B + A - 1 = 0$

$\underline{\underline{B = -1}}$ (5)

$x \rightsquigarrow B - A + 2 = a$

$-1 - 2 + 2 = a$

$\underline{\underline{a = \cancel{5}1}}$ (5)

constant $\rightsquigarrow -B + 3 = b$

$\underline{\underline{b = 4}}$ (5)

$\boxed{25}$

(04) $\lim\limits_{x \to \pi} \dfrac{\sqrt{2+\cos x} - 1}{(\pi - x)^2}$ let $\pi - x = \theta$.

$= \lim\limits_{\theta \to 0} \dfrac{\sqrt{2+\cos(\pi-\theta)} - 1}{\theta^2}$

$= \lim\limits_{\theta \to 0} \dfrac{\sqrt{2-\cos\theta} - 1}{\theta^2}$ (5)

$= \lim\limits_{\theta \to 0} \dfrac{2-\cos\theta - 1}{\theta^2\left[\sqrt{2-\cos\theta}+1\right]}$ (5)

$= \lim\limits_{\theta \to 0} \dfrac{1-\cos\theta}{\theta^2\left[\sqrt{2-\cos\theta}+1\right]}$

4

(06) $y = x^2 - 2x + 5$.

$\frac{dy}{dx} = 2x - 2$ 5

$m = \left(\frac{dy}{dx}\right)_{x=2}$

$= \underline{\underline{2}}$ 5

When $x = 2$

$y = 4 - 4 + 5$

$= 5$

$p \equiv (2, 5)$ 5

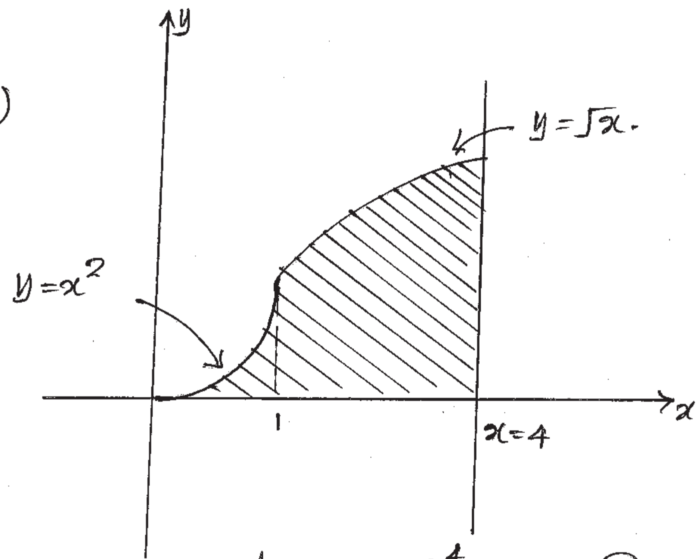
Equation of the tangent

$\frac{y-5}{x-2} = 2$

$\underline{\underline{y - 2x - 1 = 0}}$ 10

$\boxed{25}$

(07)



Area $= \int_0^1 x^2 dx + \int_1^4 \sqrt{x}\, dx$ 10

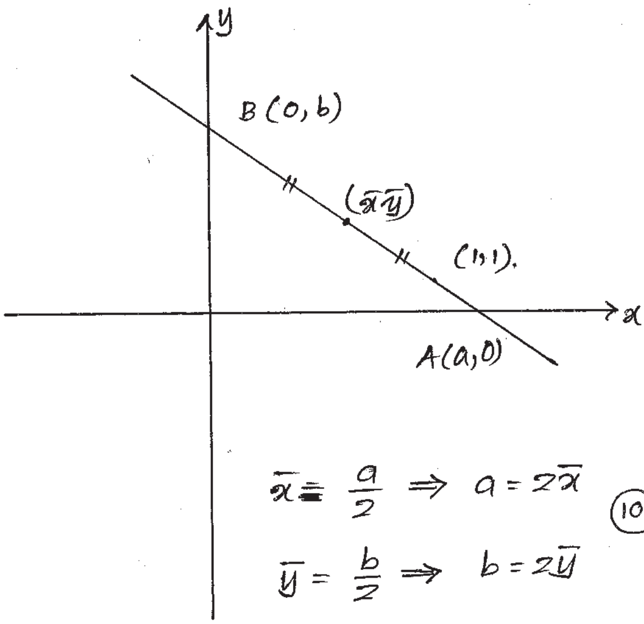
5

$\equiv \left(\frac{x^3}{3}\right)_0^1 + \left(\frac{x^{3/2}}{3/2}\right)_1^4 \equiv \frac{1}{3} + \frac{16}{3} - \frac{2}{3} = \underline{\underline{5}}$

$\equiv$ 5 5 5

$\boxed{25}$

(08)



$\bar{x} = \frac{a}{2} \Rightarrow a = 2\bar{x}$ (10)

$\bar{y} = \frac{b}{2} \Rightarrow b = 2\bar{y}$

$$\frac{b-0}{0-a} = \frac{1-0}{1-a}$$

$$-\frac{b}{a} = \frac{1}{1-a}$$

$$b - ab = -a$$

$$a + b - ab = 0 \quad (10)$$

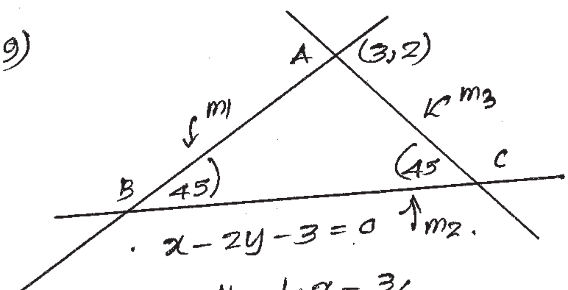
$$2\bar{x} + 2\bar{y} - 4\bar{x}\bar{y} = 0$$

$$\bar{x} \equiv x \quad \bar{y} \equiv y$$

$$x + y - 2xy = 0 \quad (5)$$

$\boxed{25}$

(09)



$$x - 2y - 3 = 0$$

$$y = \frac{1}{2}x - \frac{3}{2}$$

$$m_2 = \frac{1}{2}. \quad (5)$$

6

$$\tan 45 = \left| \frac{m_1 - \frac{1}{2}}{1 + \frac{m_1}{2}} \right|$$

$$1 = \left| \frac{2m_1 - 1}{2 + m_1} \right|$$

$$\frac{2m_1 - 1}{2 + m_1} = \pm 1$$

$\oplus \Rightarrow \quad 2m_1 - 1 = 2 + m_1$

$m_1 = 3$ (5)

$\ominus \Rightarrow 2m_1 - 1 = -2 - m_1$

$m_1 = -\frac{1}{3}$ (5)

$$\frac{y-2}{x-3} = 3 \qquad\qquad \frac{y-2}{x-3} = -\frac{1}{3}$$

$$y - 2 = 3x - 9 \qquad\qquad 3y - 6 = -x + 3$$

$$\underline{\underline{y - 3x + 7 = 0}} \ (5) \qquad\qquad \underline{\underline{3y + x - 9 = 0}} \ (5)$$

$\boxed{25}$

(10) $\underbrace{\tan^{-1}\left(\frac{1}{x-1}\right)}_{\alpha} - \underbrace{\tan^{-1}\left(\frac{1}{x+1}\right)}_{\beta} = \underbrace{\tan^{-1}\left(\frac{1}{2}\right)}_{\gamma}$

Let $\alpha = \tan^{-1}\left(\frac{1}{x-1}\right)$, $\beta = \tan^{-1}\left(\frac{1}{x+1}\right)$ and $\gamma = \tan^{-1}\left(\frac{1}{2}\right)$

then $\tan\alpha = \frac{1}{x-1}$, $\tan\beta = \frac{1}{x+1}$ and $\tan\gamma = \frac{1}{2}$

$$\alpha - \beta = \gamma$$

$$\tan(\alpha - \beta) = \tan\gamma$$

$$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \tan\gamma \quad (5)$$

$$\frac{\frac{1}{x-1} - \frac{1}{x+1}}{1 + \left(\frac{1}{x-1}\right)\left(\frac{1}{x+1}\right)} = \frac{1}{2} \quad (10)$$

$$\frac{x+1-x+1}{x^2-1+1} = \frac{1}{2}$$

$$\frac{2}{x^2} = \frac{1}{2}$$

$$x^2 = 4$$

$$x = \pm 2 \quad (5)$$

$$\therefore \underline{\underline{x = 2}} \quad (\because x > 0) \ (5)$$

$\boxed{25}$

Part B

(11) a. If the roots of $x^2-2x+3=0$ are α and β,

$\alpha+\beta=2$ (5) and $\alpha\beta=3$ (5)

i. $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$ (5)

$= 2^2-2\times 3$

$= -2$ (5)

ii. $\alpha^3+\beta^3 = (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ (5)

$= 2^3-3\times 3\times 2$

$= -10$ (5)

iii. $\alpha^4+\beta^4 = (\alpha^2)^2+(\beta^2)^2$

$= (\alpha^2+\beta^2)^2-2\alpha^2\beta^2$

$= (\alpha^2+\beta^2)^2-2(\alpha\beta)^2$ (10)

(iv) $\alpha^3-\beta$ and $\beta^3-\alpha$ are the roots of

$x^2-[(\alpha^3-\beta)+(\beta^3-\alpha)]x+(\alpha^3-\beta)(\beta^3-\alpha)=0$ (10)

since $\alpha^3-\beta+\beta^3-\alpha$

$= \alpha^3+\beta^3-(\alpha+\beta)$

$= -10-2$

$= -12$ (10) and

$(\alpha^3-\beta)(\beta^3-\alpha) = \alpha^3\beta^3-\alpha^4-\beta^4+\alpha\beta$

$= (\alpha\beta)^3+\alpha\beta-(\alpha^4+\beta^4)$

$= 3^3+3-(4-18)$

$= 30+14$

$= 44$ (10)

$\boxed{80}$

$x^2+12x+44=0$ (5)

i.e. $px^2+qx+r=0$; $p=1$, $q=12$ and $r=44$. (5)

9

b. If $(x-2)$ is a factor of

$P(x) = 3x^3 + Cx^2 + x - 2$

$3x^3 + Cx^2 + x - 2 = (x-2)(3x^2 + Ax + 1)$ 5

$\underline{\underline{x^2}}$ $C = A - 6$

$\underline{\underline{x}}$ $1 = 1 - 2A \Rightarrow A = 0$

$\Rightarrow C = -6$ 5

$\therefore 3x^3 - 6x^2 - x - 2 = (x-2)(3x^2+1)$

i. $\therefore$ The remaining factor of $P(x)$ is $\underline{\underline{3x^2+1}}$ 10

ii. $P(x) = (x-2)(3x^2+1) = 0$

$\Rightarrow x = 2$ or $x = \pm\sqrt{-1/3}$ 5

real 5 not real

$\therefore P(x) = 0$ has only 1 real root.

iii. $P(x) + \lambda(x-2)(3x-1) = 0$

$(x-2)(3x^2+1) + \lambda(x-2)(3x-1) = 0$

$(x-2)[3x^2 + 1 + 3\lambda x - \lambda] = 0$

$\therefore (x-2)[3x^2 + 3\lambda x + (1-\lambda)] = 0$ 10

For having 3 distinct roots for above equation, the discriminent should be +ve. 5

of $3x^2 + 3\lambda x + 1 - \lambda = 0$ 5

$\Delta = (3\lambda)^2 - 4 \times 3 \times (1-\lambda) > 0$

$9\lambda^2 + 12\lambda - 12 > 0$

$3\lambda^2 + 4\lambda - 4 > 0$

$3\lambda^2 + 6\lambda - 2\lambda - 4 > 0$

$(3\lambda - 2)(\lambda + 2) > 0$ 5

10

70

150

$+ \quad - \quad +$

$-2 \qquad 2/3$ (5)

$\lambda \in (-\infty, -2) \cup (2/3, \infty)$ (10)

$\boxed{70}$

(12) a. $f(x) = ax^2 + 2bx + c$

$g(x) = 2(ax+b)$

$F(x) = f(x) + \lambda g(x)$

$= ax^2 + 2bx + c + 2\lambda(ax+b)$

$= ax^2 + 2(b+\lambda a)x + (c + 2\lambda b)$ (10)

$\Delta = 4(b+\lambda a)^2 - 4a(c+2\lambda b)$

$= 4[b^2 + \cancel{2\lambda ab} + \lambda^2 a^2 - ac - \cancel{2\lambda ab}]$

$= 4(b^2 + \lambda^2 a^2 - ac)$ (10)

$\therefore$ In $F(x) = 0$, (5)

$\Delta = b^2 + \lambda^2 a^2 - ac$ — (A)

Since the roots of $f(x)$ is real and distinct, $4b^2 - 4 \cdot ac > 0$ — (B)

i.e. $b^2 - ac > 0$ (5)

$\therefore$ (B) $\Rightarrow \Delta = \underbrace{\lambda^2 a^2}_{(+)ve} + \underbrace{b^2 - ac}_{(+)ve}$ (5)

$\therefore b^2 + \lambda^2 a^2 - ac > 0$

$\therefore F(x) = 0$ has real and distinct roots (5)

40

b. i. Let $y = 5|x-1| + 3$ and $y = ax + b$.

when $x = 0$, $5|-1| + 3 = b$

$\Rightarrow b = 8$ (5)

when $x = 5$ $5|4| + 3 = 5a + 8$

$\Rightarrow a = 3$ (5)

10

11

(12) b. ii. $x^2-2x-24<0$ and $x^2+2x>0$

$$x^2-2x-24 = x^2-2x+1-24-1$$
$$= (x-1)^2-5^2$$
$$= (x-1-5)(x-1+5)$$
$$= (x-6)(x+4) \quad 5$$

$$x^2-2x-24<0 \Rightarrow (x-6)(x+4)<0$$

	$-\infty<x<-4$	$-4\leq x<6$	$6\leq x<\infty$
sign of $(x-6)(x+4)$	+	−	+

10

$$\therefore x \in (-4, 6) \quad 5$$

since $x^2+2x>0$

$$x(x+2)>0 \quad 5$$

	$-\infty<x<-2$	$-2\leq x<0$	$0\leq x<\infty$
sign of $x(x+2)$	+	−	+

10

$$\therefore x \in (-\infty,-2) \cup (0,\infty) \quad 5$$

$$\therefore x \in (-4,-2)\cup(0,6) \quad 5$$

45

c. $\log_a c = x$

$$\Rightarrow a^x = c$$
$$\Rightarrow \log_b a^x = \log_b c$$
$$\Rightarrow x\log_b a = \log_b c$$
$$\Rightarrow x = \frac{\log_b c}{\log_b a}$$

$$\therefore \log_a c = \frac{\log_b c}{\log_b a}$$

10

12

When $b = c$ in $\log_a c = \dfrac{\log_b c}{\log_b a}$

$$\log_a c = \frac{\log_c c}{\log_c a}$$

$$\therefore \log_a c = \frac{1}{\log_c a}$$

10

$x = \log_{2a} a$, $y = \log_{3a} 2a$, $z = \log_{4a} 3a$.

$$xyz + 1 = \log_{2a} a \times \log_{3a} 2a \times \log_{4a} 3a + 1$$

$$= \log_{2a} a \times \frac{\log_{3a} 2a}{\log_{3a} 4a} + 1 \quad 5$$

$$= \log_{2a} a \times \log_{4a} 2a + 1 \quad 5$$

$$= \frac{\log_{2a} a}{\log_{2a} 4a} + 1 \quad 5 \quad = \log_{4a} a + \log_{4a} 4a \quad 5$$

$$= \log_{4a} 4a^2 \quad 5 \quad = \log_{4a} (2a)^2 .$$

$$= 2 \log_{4a} 2a \quad 5$$

$$= 2yz \quad 5$$

35

150

13

(13) $y = f(x)$, $x = \tan\theta$

$$\frac{dx}{d\theta} = \sec^2\theta \quad (5)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} \quad (5)$$

$$\frac{dy}{dx} = \cos^2\theta \frac{dy}{d\theta} \quad (5)$$

$$\frac{d^2y}{dx^2} = \left\{\cos^2\theta \frac{d^2y}{d\theta^2} + 2\cos\theta(-\sin\theta)\frac{dy}{d\theta}\right\} \cdot \frac{d\theta}{dx} \quad (10)$$

$$\frac{d^2y}{dx^2} = \left\{\cos^2\theta \frac{d^2y}{d\theta^2} - 2\sin\theta\cos\theta \frac{dy}{d\theta}\right\} \cos^2\theta$$

$$\frac{d^2y}{dx^2} = \cos^4\theta \left\{\frac{d^2y}{d\theta^2} - 2\tan\theta \frac{dy}{d\theta}\right\} \quad (5)$$

30

$$(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + y = 0$$

$$\underbrace{(1+\tan^2\theta)^2\cos^4\theta}_{(5)} \left\{\frac{d^2y}{d\theta^2} - 2\tan\theta \frac{dy}{d\theta}\right\} + \quad (5)$$

$$2\tan\theta(1+\tan^2\theta)\cos^2\theta \frac{dy}{d\theta} + y = 0 \quad (5)$$

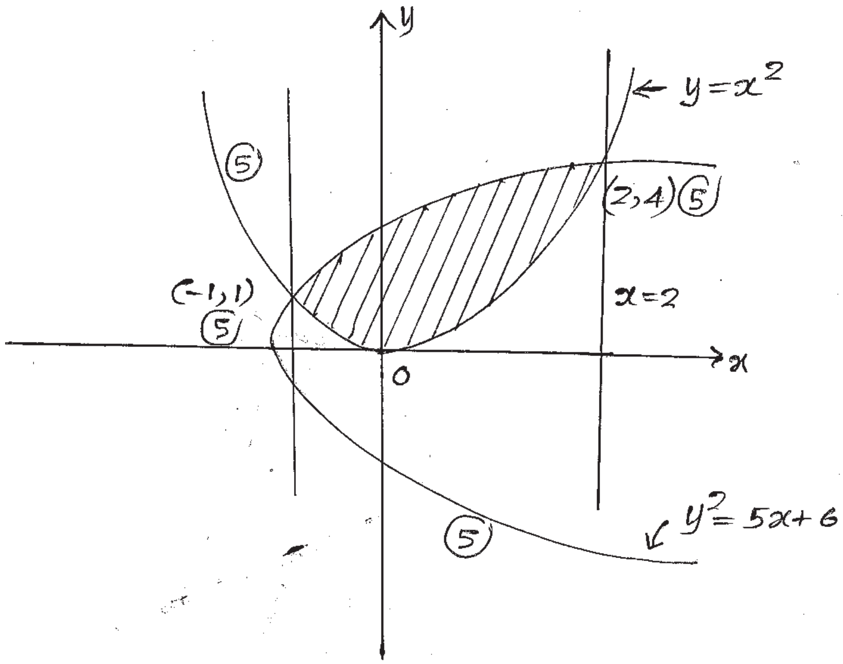
$$\frac{d^2y}{d\theta^2} - 2\tan\theta \frac{dy}{d\theta} + 2\tan\theta \frac{dy}{d\theta} + y = 0.$$

$$\frac{d^2y}{d\theta^2} + y = 0 \quad (10)$$

25

14

(b)



$$\text{Area} = \int_{-1}^{2}\sqrt{5x+6}\,dx - \int_{-1}^{2}x^2\,dx \quad 10$$

$$= \frac{2}{15}\Big[(5x+6)^{3/2}\Big]_{-1}^{2} - \left(\frac{x^3}{3}\right)_{-1}^{2} \quad 5 \quad 5$$

$$= \frac{2}{15}\left\{16^{3/2} - 1^{3/2}\right\} - \left\{\frac{8}{3}+\frac{1}{3}\right\} \quad 5 \quad 5$$

$$= \frac{2}{15}(64-1) - 3$$

$$= \frac{2}{15}\times 63 - 3$$

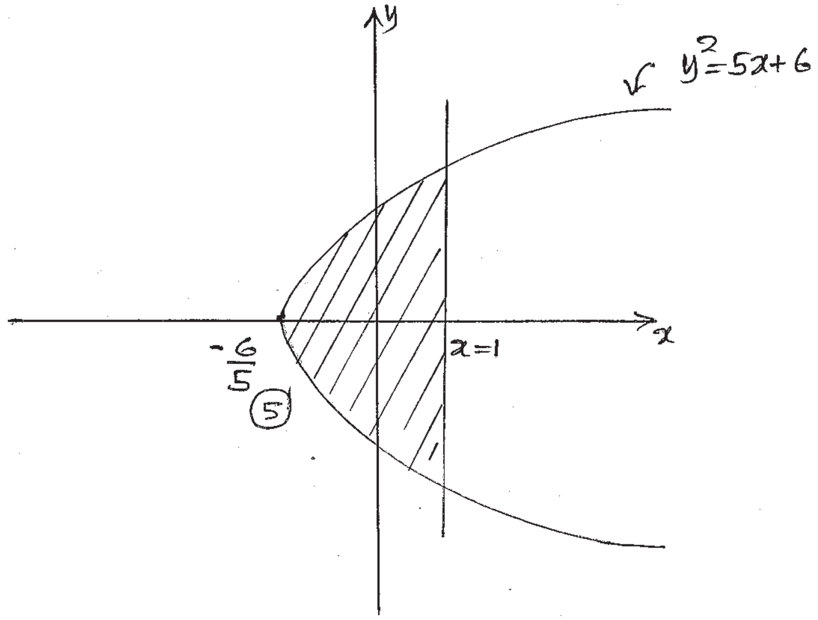
$$= \frac{126-45}{15}$$

$$= \frac{81}{15}$$

$$= 5\frac{2}{5} \quad 5$$

15

55



$$\text{Volume} = \int_{-6/5}^{1} \pi y^2 \, dx \quad 10$$

$$= \pi \int_{-6/5}^{1} 5x + 6 \, dx$$

$$= \pi \left\{ \frac{5x^2}{2} + 6x \right\}_{-6/5}^{1} \quad 10$$

$$= \pi \left[\left\{ \frac{5}{2} + 6 \right\} - \left\{ \frac{5}{2} \times \frac{36}{25} - \frac{36}{5} \right\} \right] \quad 10$$

$$= \pi \left\{ \frac{17}{2} - \frac{36}{25} \left(\frac{5}{2} - 5 \right) \right\}$$

$$= \pi \left\{ \frac{17}{2} + \frac{36}{25} \times \frac{5}{2} \right\}$$

$$= \pi \left\{ \frac{17}{2} + \frac{18}{5} \right\}$$

$$= \underline{\underline{\frac{121\pi}{10}}} \quad 5$$

40

$\boxed{150}$

16

(14) a. $f(x) = \dfrac{x+4}{(x-1)^2}$; $x \neq 1$, $x \in \mathbb{R}$.

$$f'(x) = \frac{(x-1)^2 \times 1 - (x+4) \times 2(x-1) \times 1}{(x-1)^4} \quad 10$$

$$= \frac{x-1-2x-8}{(x-1)^3} = \frac{-x-9}{(x-1)^3}$$

$$= \frac{x+9}{(1-x)^3} \quad 5$$

When $f'(x) = 0$, $x = -9$

Then $y = -1/20$

$\therefore (-9, -1/20)$ is 5 a turning point.

when $x = 1$, $f(x)$ does not exist.

$\therefore x = 1$ is a vertical asymptote 5

Since $f(x) = \dfrac{x/x^2 + 4/x^2}{x^2/x^2 - 2x/x^2 + 1/x^2} = \dfrac{1/x + 4/x^2}{1 - 2/x + 1/x^2}$

$\lim_{x \to \pm\infty} f(x) = 0$

$\therefore y = 0$ is a horizontal 5 asymptote.

First derevative test

	$-\infty < x < -9$	$-9 \leq x < 1$	$1 \leq x < \infty$
sign of $f'(x)$	$(-)$ 5	$(+)$ 5	$(-)$ 5

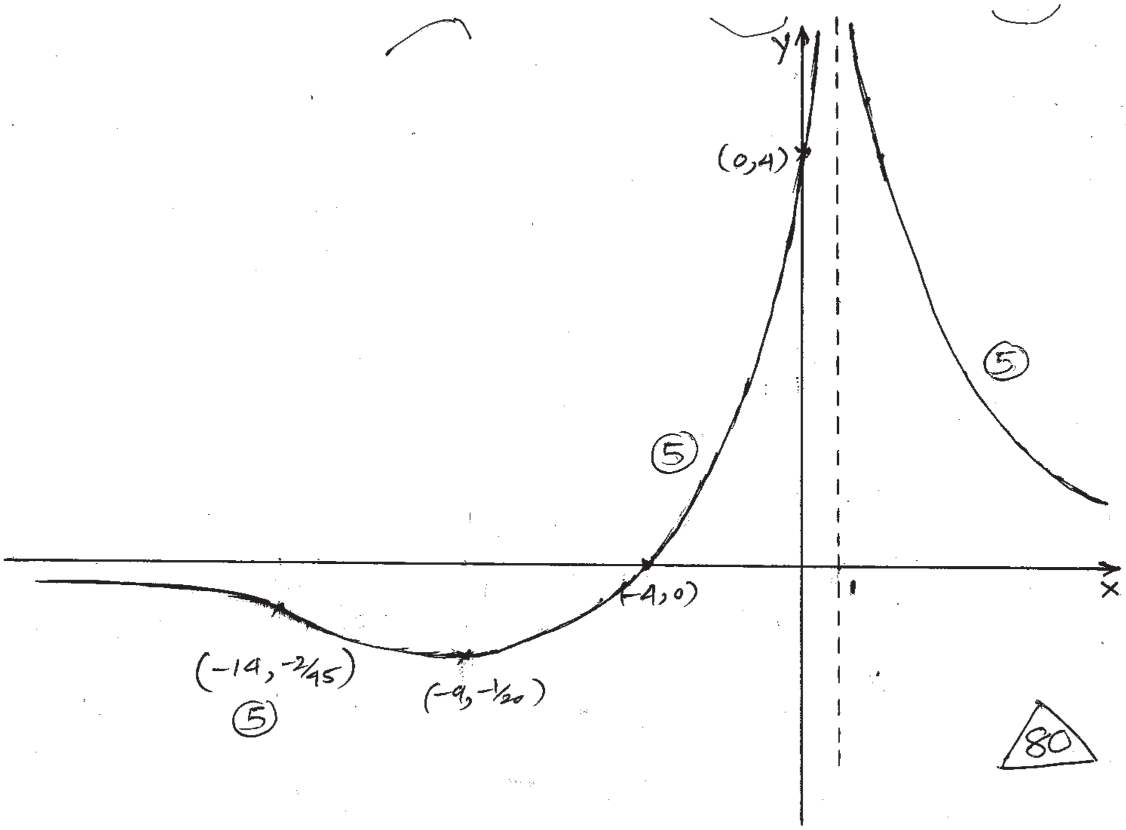
Since $f''(x) = \dfrac{2(x+14)}{(x-1)^4}$

when $x = -14$ there exist an inflection point. 5

17

second derivative test

	$-\infty < x < -14$	$-14 \le x < 1$	$1 \le x < \infty$
sign of $f''(x)$	$(-)$ concave down 5	$(+)$ concave up 5	$(+)$ concave up 5



b.

$$V = \pi R^2 H + \frac{1}{2} \times \frac{4}{3}\pi R^3 \quad 10$$

$$= \pi R^2 \left(H + \frac{2}{3}R\right)$$

$$\Rightarrow H = \frac{V}{\pi R^2} - \frac{2}{3}R = \frac{3V - 2\pi R^3}{3\pi R^2}$$

$$A = 2\pi R H + \pi R^2 + \frac{1}{2} \times 4\pi R^2 \quad 10$$

$$= \pi R(2H + 3R)$$

18

$$A = \pi R\left(\frac{6V - 4\pi R^3}{3\pi R^2} + 3R\right) \quad (5)$$

$$= \frac{\pi R\,(6V + 5\pi R^2)}{3\pi R^2}$$

$$= \frac{6V + 5\pi R^3}{3R}$$

A is a function of R.

$$\frac{dA}{dR} = \frac{3R \times 15\pi R^2 - (6V + 5\pi R^3) \times 3}{9R^2} \quad (10)$$

$$= \frac{45\pi R^3 - 18V - 15\pi R^3}{9R^2}$$

$$= \frac{10\pi R^3 - 6V}{3R^2} \quad (5)$$

When $\frac{dA}{dR} = 0$, $R = \sqrt[3]{\frac{3V}{5\pi}}$ (5)

$$\frac{d^2A}{dR^2} = \frac{3R^2 \times 30\pi R^2 - (10\pi R^3 - 6V)\cdot 6R}{9R^4}$$

$$= \frac{90\pi R^3 - 60\pi R^3 + 36V}{9R^3}$$

$$= \frac{10\pi R^3 + 12V}{3R^3} \quad (5)$$

When $R = \sqrt[3]{\frac{3V}{5\pi}}$; $\frac{d^2A}{dR^2} = \frac{10\pi \times \frac{3V}{5\pi} + 12V}{3 \times \frac{3V}{5\pi}}$

$$= \frac{18V}{9V} \times 5\pi$$

$$= 10\pi > 0 \quad (5)$$

∴ the curve is concave up. (5)

∴ when $R = \sqrt[3]{\frac{3V}{5\pi}}$, A is minimum (5)

Then $H = \frac{3V - 2\pi \cdot \frac{3V}{5\pi}}{3\pi\left(\frac{3V}{5\pi}\right)^{2/3}} = \frac{9V}{\frac{15}{5}\pi \cdot \left(\frac{3V}{5\pi}\right)^{2/3}} = \sqrt[3]{\frac{3V}{5\pi}}$ (5) 70

19

(15) a. $\dfrac{1}{(x^2-1)(2x+1)} = \dfrac{1}{(x-1)(x+1)(2x+1)}$

$$= \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+1} \quad (5)$$

$$1 = A(x+1)(2x+1) + B(x-1)(2x+1) + C(x-1)(x+1)$$

$x^2 \rightsquigarrow A + B + C = 0$

$x \rightsquigarrow 3A - B = 0$

Constant $\rightsquigarrow A - B - C = 1$

$A = \frac{1}{6} \quad B = \frac{1}{2} \quad C = \frac{2}{3}$ (15)

$$\int \frac{1}{(x^2-1)(2x+1)}\,dx = \frac{1}{6}\int\frac{1}{x-1}\,dx + \frac{1}{2}\int\frac{1}{x+1}\,dx + \frac{1}{3}\int\frac{2}{2x+3}\,dx$$
(5) (5)

$$= \frac{1}{6}\ln|x-1| + \frac{1}{2}\ln|x+1| + \frac{1}{3}\ln|2x+3| + c.$$
(5) (5) (5) (5)

50

Substitute $x = \cos\theta$

$\dfrac{dx}{d\theta} = -\sin\theta$ (5)

$$\int \frac{1}{(\cos^2\theta-1)(2\cos\theta+1)}(-\sin\theta)\,d\theta =$$
(5)

$$\frac{1}{6}\ln|\cos\theta-1| + \frac{1}{2}|\cos\theta+1| + \frac{1}{3}\ln|2\cos\theta+3| + c$$
(5)

20

$$\int \frac{1}{\sin\theta(2\cos\theta+1)}\,d\theta = \frac{1}{6}\ln|\cos\theta-1| + \frac{1}{2}\ln|\cos\theta+1| + \frac{1}{3}\ln|2\cos\theta+3| + C$$

$$\int \frac{1}{\sin\theta+\sin 2\theta}\,d\theta = \frac{1}{6}\ln|\cos\theta-1| + \frac{1}{2}\ln|\cos\theta+1| + \frac{1}{3}\ln|2\cos\theta+3| + C$$

5

20

(b) $$\int e^{5x}\sin 2x\,dx = \int \sin 2x \frac{d}{dx}\left(\frac{e^{5x}}{5}\right)dx$$ 5

$$= \frac{e^{5x}}{5}\sin 2x - \int \frac{e^{5x}}{5}\cos 2x \cdot 2\,dx$$ 5

$$= \frac{e^{5x}}{5}\sin 2x - \frac{2}{5}\int \cos 2x \frac{d}{dx}\left(\frac{e^{5x}}{5}\right)dx$$ 5

$$= \frac{e^{5x}}{5}\sin 2x - \frac{2}{5}\left\{\frac{e^{5x}}{5}\cos 2x - \int \frac{e^{5x}}{5}\sin 2x \cdot 2\,dx\right.$$ 5

$$= \frac{e^{5x}}{5}\sin 2x - \frac{2}{25}e^{5x}\cos 2x + \frac{4}{25}\int e^{5x}\sin 2x\,dx$$

$$\frac{21}{25}\int e^{5x}\sin 2x\,dx = \frac{e^{5x}}{25}(5\sin 2x - \cos 2x)$$ 5 5

$$\int e^{5x}\sin 2x\,dx = \frac{e^{5x}}{21}(5\sin 2x - \cos 2x) + C.$$

30

21

c.) $\displaystyle\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^2 x + \sin^4 x}\,dx = \frac{1}{2}\int_0^{\pi/4} \frac{2\sin x \cos x}{\cos^2 x + \sin^2 x\,(1-\cos^2 x)}\,dx$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{\sin 2x}{\cos^2 x + \sin^2 x - \sin^2 x \cos^2 x}\,dx$$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{\sin 2x}{1 - \sin^2 x \cos^2 x}\,dx$$

$$= \frac{4}{2}\int_0^{\pi/4} \frac{\sin 2x}{4 - \sin^2 2x}\,dx$$

$$= 2\int_0^{\pi/4} \frac{\sin 2x}{4 - 1 + \cos^2 2x}\,dx$$

$$= 2\int_0^{\pi/4} \frac{\sin 2x}{\cos^2 2x + 3}\,dx$$

15

$z = \cos 2x$

$\dfrac{dz}{dx} = -2\sin 2x$ (5)

When $x = 0$, $z = \cos 0 = 1$ (5)

When $x = \pi/4$, $z = \cos \pi/2 = 0$.

$$\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^2 x + \sin^4 x}\,dx = 2\int_0^{\pi/4} \frac{\sin 2x}{\cos^2 2x + 3}\,dx \quad (5)$$

$$= 2\int_1^0 \frac{-1}{z^2+3}\,dz$$

$$= \int_0^1 \frac{1}{z^2+3}\,dz \ (5) = \frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{z}{\sqrt{3}}\right)\right]_0^1 \ (5)$$

35

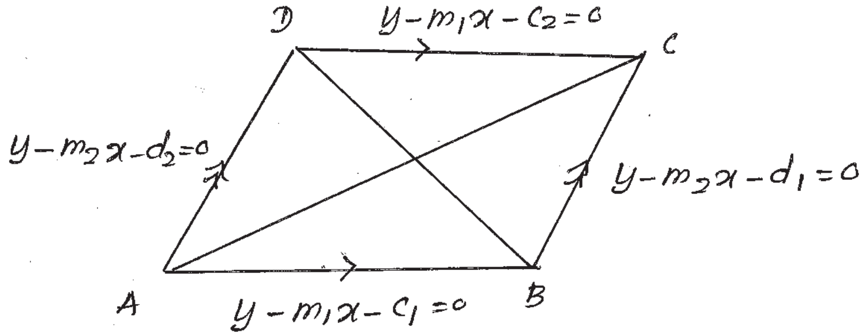
(5)

$$= \frac{1}{\sqrt{3}}\left\{\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}(0)\right\} \ (5) = \frac{1}{\sqrt{3}}\cdot\frac{\pi}{6} = \frac{\pi}{6\sqrt{3}}$$

22

(16) Theory

$$(a_1x+b_1y+c_1)+\lambda(a_2x+b_2y+c_2)=0.$$ 20



AC

The equation of any straight line passes through C ;

$$(y-m_1x-c_2)+\lambda(y-m_2x-d_1)=0 \quad ; \lambda \text{ is a Parameter.}$$

let $A(\bar{x}\,\bar{y})$

Finding λ such that it passes through $A(\bar{x}\,\bar{y})$.

$$(\bar{y}-m_1\bar{x}-c_2)+\lambda(\bar{y}-m_2\bar{x}-d_1)=0 \quad -(A).$$

Since $A(\bar{x}\,\bar{y})$ is on the lines AB and AD.

$$\bar{y}-m_1\bar{x}-c_1=0 \Rightarrow \bar{y}-m_1\bar{x}=c_1 \quad -(1)$$

$$\bar{y}-m_2\bar{x}-d_2=0 \Rightarrow \bar{y}-m_2\bar{x}=d_2 \quad -(2)$$

Substitute (1) and (2) in (A).

$$(c_1-c_2)+\lambda(d_2-d_1)=0$$

23

$$\lambda = \left(\frac{c_2 - c_1}{d_2 - d_1} \right)$$

$\therefore$ The equation of AC

$$(y - m_1 x - c_2) + \left(\frac{c_2 - c_1}{d_2 - d_1} \right) (y - m_2 x - d_1) = 0 \qquad 30$$

BD

Simillarly.

$$(y - m_1 x - c_2) + \left(\frac{c_2 - c_1}{d_1 - d_2} \right) (y - m_2 x - d_2) = 0. \qquad 10$$

AC

$$\left(1 + \frac{c_2 - c_1}{d_2 - d_1}\right) y - \left[m_1 + m_2 \left(\frac{c_2 - c_1}{d_2 - d_1} \right) \right] x - c_2 - d_1 \left(\frac{c_2 - c_1}{d_2 - d_1} \right) = 0$$

$$(d_2 - d_1 + c_2 - c_1) y - \{ m_1 (d_2 - d_1) + m_2 (c_2 - c_1) \} x - c_2 (d_2 - d_1) - d_1 (c_2 - c_1) = 0$$

$$M_1 = \frac{m_1 (d_2 - d_1) + m_2 (c_2 - c_1)}{(d_2 - d_1 + c_2 - c_1)} \qquad 10$$

BD

$$M_2 = \frac{m_1 (d_1 - d_2) + m_2 (c_2 - c_1)}{(d_1 - d_2 + c_2 - c_1)} \qquad 10$$

24

If $ABCD$ is a rhombus, then $AC \perp BD$

$$\Rightarrow M_1 M_2 = -1 \quad 5$$

5

$$\frac{m_1(d_2-d_1)+m_2(c_2-c_1)}{(d_2-d_1+c_2-c_1)} \cdot \frac{m_1(d_1-d_2)+m_2(c_2-c_1)}{(d_1-d_2+c_2-c_1)} = -1$$

$$\frac{\{m_1(d_2-d_1)+m_2(c_2-c_1)\}\{-m_1(d_2-d_1)+m_2(c_2-c_1)\}}{[(d_2-d_1)+(c_2-c_1)][-(d_2-d_1)+(c_2-c_1)]} = -1$$

$$m_2^2(c_2-c_1)^2 - m_1^2(d_2-d_1)^2 = -\{(c_2-c_1)^2-(d_2-d_1)^2\}$$

$$\underline{\underline{(1+m_1^2)(d_2^2-d_1^2) = (1+m_2^2)(c_1-c_2)^2}} \quad 20$$

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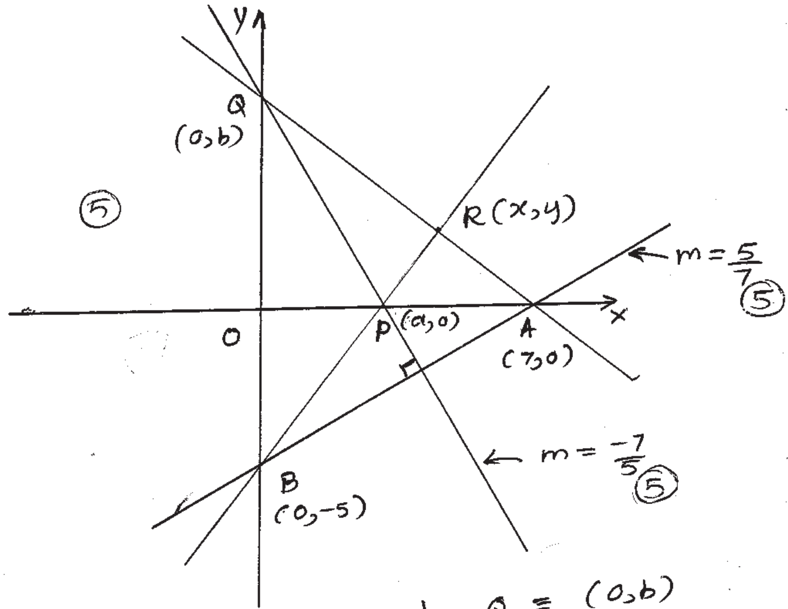
$\boxed{100}$

25

(16) b.

5



Let $P \equiv (a, 0)$ and $Q \equiv (0, b)$

Then $\frac{b}{-a} = \frac{-7}{5} \Rightarrow b = \frac{7}{5}a$ 5

For straight line BPR

$\frac{y+5}{x} = \frac{5}{a} \Rightarrow a = \frac{5x}{y+5}$ — A 10

For straight line AQR

$\frac{y}{x-7} = \frac{b}{-7} = \frac{7}{5}a \times \frac{1}{-7} = \frac{-a}{5}$

$\Rightarrow -a = \frac{5y}{x-7}$ — B 10

A + B $\Rightarrow \frac{5x}{y+5} + \frac{5y}{x-7} = 0$ 5

$5x^2 - 35x + 5y^2 + 25y = 0$

$x^2 + y^2 - 7x + 5y = 0$ 5

The equation of the locus of the point R

50

26

(17) a. $\tan(A-B) = \dfrac{\tan A - \tan B}{1-\tan A \tan B}$ 5

When $A = 45^\circ$ and $B = 30^\circ$

5

$$\tan(45^\circ - 30) = \frac{\tan 45^\circ - \tan 30^\circ}{1-\tan 45^\circ \tan 30^\circ}$$

$$\tan 15^\circ = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$ 5

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}-1)^2}{3-1}$$

$$= \frac{3-2\sqrt{3}+1}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \underline{\underline{2-\sqrt{3}}}$$ 5

20

$$\tan\left(\frac{x}{2}\right) = \frac{\sqrt{1+\tan^2 x}-1}{\tan x}$$

$$RHS = \frac{\sqrt{1+\tan^2 x}-1}{\tan x}$$

$$= \frac{\sqrt{\sec^2 x}-1}{\tan x}$$

$$= \frac{\sec x - 1}{\tan x}$$

27

$$= \frac{1-\cos x}{\sin x}$$

$$= \frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$$

$$= \frac{\sin\left(x/2\right)}{\cos\left(x/2\right)}$$

$$= \tan(x/2)$$

10

$\therefore$ LHS = RHS

$$\tan\left(x/2\right) = \frac{\sqrt{1+\tan^2 x}-1}{\tan x}$$

When $x = 15^\circ$

$$\tan\left(7\tfrac{1}{2}^\circ\right) = \frac{\sqrt{1+\tan^2 15}-1}{\tan 15^\circ}$$

$$= \frac{\sqrt{1+\left(2-\sqrt{3}\right)^2}-1}{2-\sqrt{3}} \quad 5$$

$$= \frac{\sqrt{8-4\sqrt{3}}-1}{2-\sqrt{3}}$$

$$= \frac{\sqrt{\left(\sqrt{6}-\sqrt{2}\right)^2}-1}{2-\sqrt{3}} \quad 5$$

$$= \frac{\sqrt{6}-\sqrt{2}-1}{2-\sqrt{3}}$$

$$= \left(\sqrt{6}-\sqrt{2}-1\right)\left(2+\sqrt{3}\right)$$

28

$$= 2\sqrt{6} - 2\sqrt{2} - 2 + 3\sqrt{2} - \sqrt{6} - \sqrt{3}$$

$$= \sqrt{6} + \sqrt{2} - \sqrt{3} - 2$$

$$= (\sqrt{3} - \sqrt{2})(\sqrt{2} - 1) \quad 5$$

15

$$\cot\left(7\tfrac{1}{2}^\circ\right) = \frac{1}{\tan\left(7\tfrac{1}{2}^\circ\right)}$$

$$= \frac{1}{(\sqrt{3} - \sqrt{2})(\sqrt{2} - 1)} \quad 5$$

$$= (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1)$$

$$= \sqrt{6} + \sqrt{3} + \sqrt{4} + \sqrt{2}. \quad 5$$

10

(b) $\cot x + \operatorname{cosec} x = \sqrt{3}$

$$\frac{1 + \cos x}{\sin x} = \sqrt{3}$$

$$\cos x - \sqrt{3}\sin x = -1 \quad 5$$

$$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = -\frac{1}{2}$$

$$\cos \pi/3 \cos x - \sin \pi/3 \sin x = -\frac{1}{\sqrt{2}}$$

5

$$\cos\left(x + \pi/3\right) = -\cos \pi/3$$

5

$$\cos\left(x + \pi/3\right) = \cos 2\pi/3$$

$$x + \pi/3 = 2n\pi \pm \frac{2\pi}{3}$$

20

5

$x = 2n\pi + \frac{\pi}{3}$; $n \in \mathbb{Z}$ or $x = 2n\pi - \pi$; $n \in \mathbb{Z}$

c. Theorem. 25

$$(b-c)\cos\left(\frac{A}{2}\right) = a\sin\left(\frac{B-C}{2}\right)$$

let $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = K$

Then :; $a = K\sin A$

$b = K\sin B$

$c = K\sin C$

$$\frac{b-c}{a} = \frac{K\sin B - K\sin C}{K\sin A}$$

5

$$= \frac{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{\sin A}$$

$$= \frac{2\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\sin\left(\frac{B-C}{2}\right)}{\sin A}$$

5

$$= \frac{2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B-C}{2}\right)}{2\sin\left(A/2\right)\cos\left(A/2\right)}$$

5

$$(b-c)\cos\left(A/2\right) = a\sin\left(\frac{B-C}{2}\right)$$

15

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(ii)

$$\frac{a+b-c}{a+b+c} = \frac{K\sin A + K\sin B - K\sin C}{K\sin A + K\sin B + K\sin C}$$

5

$$= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \sin C}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \sin C}$$

5

$$= \frac{2\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - \sin C}{2\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + \sin C}$$

5 5

$$= \frac{2\cos\left(\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}{2\cos\left(\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + 2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$= \frac{\cos\left(\frac{A-B}{2}\right) - \sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right)}{\cos\left(\frac{A-B}{2}\right) + \sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right)}$$

5

$$= \frac{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)}$$

5

$$= \frac{2\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}{2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)}$$

5

$$= \tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)$$

35

150

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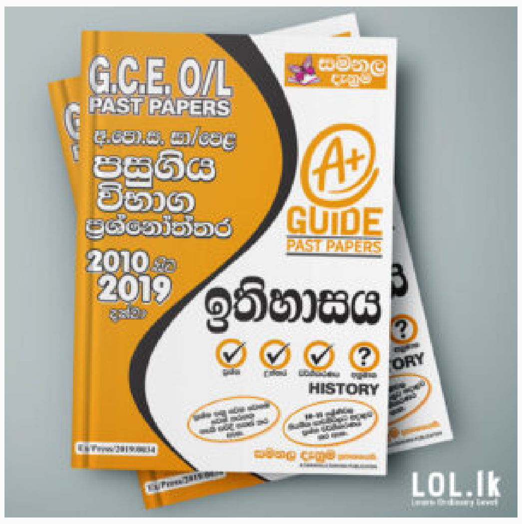

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